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Estimation of Aircraft Fuel Consumption

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Fuel conservation in civil aviation is becoming more critical. Methods are needed to estimate the impact of current or proposed air traffic control (ATC) procedures. Previous methods rely on detailed models of aircraft dynamics or on a statistical data base of past experiences. These methods are not entirely suitable for analysis of ATC procedures. MITRE has developed an algorithm for estimating the fuel consumption of commercial jet aircraft from path profile data. This permits easy evaluation of new ATC procedures without laborious and expensive computer programs or real-time testing. The approach to this algorithm is detailed in this paper. It derives from basic concepts of energy balance. It is based upon generally known assumptions and approximations for commercial jet operations. The algorithm has been used successfully in several studies of ATC procedures. Detailed verification by comparison to actual aircraft radar tracking and cockpit instrument data is underway.

Nomenclature

A	= constant, s
$\bar{a}$	= average acceleration, ft/s^2
b	= constant, s/ft
B	= fuel burn (or consumption), lb
C_D	= drag coefficient, dimensionless
C_L	= lift coefficient, dimensionless
E_D	= drag work, ft-lb
E_T	= thrust work, ft-lb
$f_i(\cdot)$	= "function of"
F_n	= thrust, lb
F_n/W_f	= reciprocal thrust specific fuel consumption = 1/TSFC, lb/lb/s
g	= gravitation acceleration = 322, ft/s^2
G	= gradient, dimensionless
h_n	= altitude, $(n=1,2)$, ft
K_n	= constant, $(n=1,2)$, dimensionless
KE	= kinetic energy, ft-lb
L	= lift, lb
M	= Mach number, dimensionless
PE	= potential energy, ft-lb
S_w	= wing area, ft^2
T	= elapsed time during an increment, s
TSFC	= thrust specific fuel consumption, lb/s
u	= velocity $(V^2 = V_I^2 + 2ax)$, ft/s
V_{Tn}	= true velocity, ft/s
$\bar{V}$	= average velocity, ft/s
W	= aircraft weight, lb
W_f	= fuel flow, lb/s
$W_f T$	= fuel burn = B, lb
X	= distance traveled, ft
z	= constant, s
β	= constant, s/ft^2
γ	= flight-path angle, rad
δ	= constant, s/ft
θ	= atmospheric constant, dimensionless
$\bar{\rho}$	= atmospheric density (average), slugs/ft^3

Introduction

THE air transportation industry is today faced with the challenge of maintaining a viable economic position in the face of unparalleled increases in turbojet fuel costs in combination with the everpresent threat of supply shortages. The close coupling between profitability and fuel usage can be illustrated by considering that (based upon mid-1979 typical air carrier cost performance) a 3% change in fuel use translates into a 23% change in profits.

The key elements that have the major effect on fuel consumption are aircraft design, operating methods, and the air traffic control environment.

Existing aircraft design is fixed, with the exception of minor modifications and specialized maintenance that can achieve an estimated potential 3% reduction in fuel consumption. Operating methods have an effect on fuel consumption in both in-flight management and preflight planning. Current experience with flight management computers indicates savings in the range of 2-4%, with an additional 4% potential savings from a preflight planning capability. Thus it appears the potential for saving fuel by changes in operating methods ranges from 2-8%.[1,2]

The final element, the Air Traffic Control environment, is the most important because it can impact fuel consumption to a greater extent than the fuel savings potential of the first two elements. As an example, a B-727 aircraft will burn approximately 16,000 lb of fuel during a 740-n.mi. trip. By assuming that the aircraft operators can achieve a 7% fuel savings during the trip by minor aircraft modifications, specialized maintenance procedures, and the use of performance computers, a saving of about 1100 lb of fuel can be achieved. However, if the aircraft suffers a delay of only 7 min in the terminal area (i.e., 7 min × 180 lb/min); then approximately 1260 lb of additional fuel will be burned. Thus all the fuel savings obtained by elements 1 and 2 are offset and an additional 160 lb are consumed. This simple case highlights the important role that air traffic control (ATC) plays in fuel conservation, since delays much greater than 7 min are not uncommon. Reducing the amount of fuel consumed owing to delay involves knowing the effect of alternate profiles and procedures on fuel consumption for the various aircraft types operating in the system; i.e., a penalty in one area may be offset by benefits in other areas for a net benefit in the system.

Efforts are underway within the FAA and the aviation community to provide more fuel efficient operations. Critical to many of these efforts is the ability to estimate the fuel efficiency of alternative procedures. MITRE, under FAA auspices, has developed a fuel consumption algorithm for this purpose. It is designed to accept static constants relating to aircraft performance, and dynamic inputs describing the profile actually flown. From this, fuel consumption can be estimated.

The fuel energy consumed by a turbojet aircraft as it traverses a path profile is directly related to the energy gains and losses. This energy balance can be expressed as

$$\text{(energy in)} - \text{(energy loss)} = \text{(energy change)}$$

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or, when expressed in terms of aircraft physical variables,

$$f(E_T) - f(E_D) = f(\Delta \mathrm{KE}) + f(\Delta \mathrm{PE})$$

During the aircraft flight, the pilot selects the desired changes in kinetic and potential energies, suffers the loss in energy resulting from drag changes, and adjusts the amount of thrust energy expended to maintain the energy balance. Thus the aircraft flight can be characterized as a system of energy losses and gains that must be continuously balanced by changes in the consumption of fuel energy.

The algorithm that has been developed uses this approach, based only upon the descriptions of the aircraft configuration, weight, and path profile. Thrust values are implicit within the model, rather than required as input. This permits a straightforward derivation and evaluation of the algorithm in the ATC context. This approach differs from other efforts in estimating fuel consumption. Airline calculations are intended to forecast fuel supply requirements. These are based on city pair historical use data, and therefore lack the detail necessary for ATC considerations. Air carrier flight planning is performed by utilizing an incremental table look-up routine that provides typical flight profiles. Again, the level of detail is not adequate for ATC analysis use. On the other hand, simulation research uses detailed equations of motion.[3-5] These contain thrust and other variables as an explicit input variable. As such they are difficult to calibrate and verify for use with ATC system analyses.

Assumptions

An aircraft path profile can be described by considering the changes in velocity or true airspeed (V), altitude (h), and time (T). In order to derive the simplest form of the energy balance equation, the total path profile is divided into increments of approximately 2000-ft altitude changes in the case of climbs and descents, or approximately 200 s during level flight. It has been found that this size increment yields the level of accuracy desired, when using both actual and performance handbook data. Other increments can be chosen, based upon accuracy requirements of a particular application. Additionally, use of increments permits the following simplifying assumptions:

1) An average atmospheric density ($\bar{\rho}$) is used for calculations over the increment.

2) The aircraft weight change over an increment is small as compared to the total weight, therefore an average weight (W) is used for incremental calculations.

3) The acceleration during an increment is constant ($\bar{a}$).

4) The flight-path angle (γ) is small, therefore $\cos\gamma \cong 1$, or the aircraft weight equals the required lift.

5) The climb/descent rates are linear in an increment, thus $h_2 = h_1 + GX$.

6) Upper wind effects on fuel consumption are not a part of the computational requirement; therefore the velocity/true air speed (V) equals the ground speed.

7) The standard ICAO atmospheric conditions would apply, thereby permitting the density ($\bar{\rho}$) variation with altitude (h) to be calculated by using the following equations:

$$\bar{\rho} = 0.002377\,[1 - 3.446 \times 10^{-6}(h_1 + h_2)]^{4.2439}$$

for altitudes equal to or less than 36,089 ft, and

$$\bar{\rho} = 7.062 \times 10^{-4} \exp[1.7349 - 2.404 \times 10^{-5}(h_1 + h_2)]$$

for altitudes greater than 36,089 ft.

8) The functional forms used for lift vs drag, and thrust over fuel flow, are sufficient to obtain a good data fit over the desired speed range.

Functional Relationships

The basic energy balance equation, as previously described, can be written as

$$E_T = \Delta\mathrm{KE} + \Delta\mathrm{PE} + E_D$$

Each energy term can be related to the aircraft performance and path profile variables as follows:

$$\Delta\mathrm{KE} = f_1(V_1, V_2, W_1, W_2) \tag{1}$$

$$\Delta PE = f_2(h_1, h_2, W_1, W_2) \tag{2}$$

$$E_D = f_3(\bar{\rho}, V, S_w, C_D, X) \tag{3}$$

In the above relationship, all of the variables are known path profile dependent except the wing area (S_w) and the drag coefficient (C_D). The wing area (S_w) is, of course, aircraft specific and is known for each aircraft type. However, in the case of C_D, there exists a unique relationship for a particular aircraft and configuration with a specific lift coefficient (C_L). This means that the lift coefficient (C_L) must be obtained under a specific set of operating conditions that will provide the required lift; after which the C_D can be determined. This process can be described as follows:

$$C_L = f_4(W, \bar{\rho}, S_w, V) \tag{4}$$

The drag coefficient can then be written in terms of the lift coefficient as

$$C_D = f_5(C_L^2) \tag{5}$$

By substituting f_4 into f_5, the drag coefficient can be described as

$$C_D = f_6(W, \bar{\rho}, S_w, V) \tag{6}$$

Combining f_6 with f_3, the drag work can be written as

$$E_D = f_7(V, W, \bar{\rho}, S_w, X) \tag{7}$$

The density ($\bar{\rho}$) over the increment is expressed as

$$\bar{\rho} = f_8(h) \tag{8}$$

The incremental distance (X) can be expressed in terms of the time (T) and velocity (V) as

$$X = f_9(T, V) \tag{9}$$

Based upon the restriction of the solution to small increments and the constant acceleration assumption,

$$X = \frac{V_1 + V_2}{2}T$$

Therefore by combining f_3, f_8, and f_9 the drag work can be expressed as

$$E_D = f_{10}(V, W, h, S_w, T) \tag{10}$$

This basic energy balance equation can now be expressed as a function of known variables that are either aircraft-type performance parameters, or path/profile variables. However, to calculate the estimated fuel consumption a relationship is needed between fuel burn and thrust work.

The engine fuel flow rate (W_f) that is required to produce the necessary thrust can be expressed as a function of the altitude and velocity. Generalized turbojet engine performance data indicate that the thrust (F_n) can be expressed as

$$F_n = f_{11}(V, h, W_f) \tag{11}$$

From the basic definition of work, the thrust energy can be written as

$$E_T = f_{12}(F_n, X) \tag{12}$$

Therefore, substituting f_{11} into f_{12}, the thrust energy can be described as

$$E_T = f_{13}(V, h, T, W_f) \tag{13}$$

In the above function, fuel consumption can be expressed as

$$B = f_{14}(T, W_f) \tag{14}$$

The thrust energy relationship now becomes (assuming separability)

$$E_T = f_{15}(V, h) f_{14}(T, W_f) \tag{15}$$

By combining the functional relationships (1-15) into the energy balance equation

$$E_T = E_D + \Delta PE + \Delta KE$$

fuel consumption can be derived:

$$B = f_{14}(T, W_f) = \frac{f_{10}(V, W, h, S_w, T) + f_2(h, W) + f_1(V, W)}{f_{15}(V, h)}$$

This can be written functionally as

$$B = f_{14}(T, W_f) = f_{16}(V, W, h, S_W, T) \tag{16}$$

Functions (1-15) all represent well-established standard aerodynamic equations, except the thrust-fuel flow relationship:

$$F_n = f_{11}(V, h, W_f)$$

The thrust specific fuel consumption (TSFC) curves are available from manufacturer's data that represent the pounds of fuel that must be burned per hour to produce 1 lb of thrust.

Algorithm Derivation

The derivation of the fuel consumption algorithm can now be made, based upon the detailed derivations in Refs. 2, 6, and 7. The organization is intended to parallel that of the functional relationships presented above. Thus equation numbers for the functional relationships will be suffixed with an "a" in this section to denote their relationship.

It is pertinent to note that a great deal of the performance data concerning both an airframe and an engine represents an approximation of how a typical machine will function. These approximations represent the blending of theoretical, wind tunnel, and some actual flight test results. This does not imply the performance data are exact for every aircraft of that type. However, it does imply that the accuracy of a model is largely determined by the ingenuity and effort that is exercised in determining the appropriate mathematical functions based upon the performance data. These functions are then calibrated and verified by utilizing actual flight test data.

In summary, the task is to determine empirical relationships that will describe the aircraft's operation with a satisfactory degree of accuracy over the variations in operating conditions that will be encountered.

The generalized energy balance equation can be written as

$$E_T = \Delta KE + \Delta PE + E_D$$

The integral form of this equation is

$$\int_0^X F_n \mathrm{d}x = \Delta KE + \Delta PE + \int_0^X D \mathrm{d}x$$

Basic Relationships

The change in kinetic energy may be denoted by

$$\Delta KE = W(V_2^2 - V_1^2)/2g \tag{1a}$$

The change in potential energy may be denoted by

$$\Delta PE = W(h_2 - h_1) \tag{2a}$$

Certain other basic relationships are useful and are now listed. (These relationships are well-known aerodynamic or physical relationships and can be found in Ref. 1 or 8.)

$$L = 0.5\bar{\rho} S_w V^2 C_L \quad \text{(lift)}$$

$$D = 0.5\bar{\rho} S_w V^2 C_D \quad \text{(drag)}$$

$$L = W\cos\gamma \approx W \quad (\text{since } \cos\gamma \approx 1 \text{ is assumed}).$$

The assumption of a constant average acceleration and small increments allows the computation of $\bar{a}$ and X as follows:

$$\bar{a} = \frac{V_2 - V_1}{T} \qquad X = \frac{V_1 + V_2}{2} T$$

Drag Work

Since lift is approximately equal weight,

$$L = W = 0.5\bar{\rho} S_w V^2 C_L$$

(from Ref. 1, 8); and solving for C_L,

$$C_L = W/0.5\bar{\rho} S_w V^2 \tag{4a}$$

Also, the drag is (from Ref. 8)

$$D = 0.5\bar{\rho} S_w V^2 C_D$$

From Refs. 1 and 8,

$$C_D = K_1 + K_2 C_L^2 \dagger \tag{5a}$$

By substituting for C_D,

$$D = K_1 \bar{\rho} S_w V^2/2 + 2K_2 W^2/\bar{\rho} S_w V^2$$

The drag work term thence becomes

$$E_D = \int_0^X D\mathrm{d}x$$

Noting that $\mathrm{d}x = V\mathrm{d}t$, this equals

$$\int_0^T DV\mathrm{d}t$$

Substituting for D, noting

$$V = V_1 + \bar{a}t$$

†This relationship has proven adequate for moderate C_L's and velocities encountered during transport aircraft climb, cruise, and descent operations. However, during takeoff and initial climb, K_1 and K_2 multipliers are utilized as part of the configuration (gear and flap) effect on aircraft drag.

Integrating, and simplifying, we have

$$\int_0^X D\mathrm{d}x = \int_0^T DV\mathrm{d}t = \int_0^T \left[\frac{K_1}{2}\bar{\rho}S_w V^3 + \frac{2K_2 W^2}{S_w V}\right]\mathrm{d}t$$

$$= \frac{K_1}{8}\bar{\rho}S_w T(V_1+V_2)(V_1^2+V_2^2) + \frac{2K_2 W^2 T}{\bar{\rho}S_w(V_2-V_1)}\log\frac{V_2}{V_1}$$

Since $(V_1+V_2)^2$ dominates $(V_1-V_2)^2$, we have the following approximations:

$$\frac{\log V_2/V_1}{V_2-V_1} = \frac{2}{V_1+V_2}$$

and

$$(V_1+V_2)(V_1^2+V_2^2) = (V_1+V_2)^3/2$$

Thus

$$E_D = \frac{K_1}{16}\bar{\rho}S_w T(V_1+V_2)^3 + \frac{4K_2 W^2 T}{\bar{\rho}S_w(V_1+V_2)} \tag{10a}$$

Thrust Work

An empirical equation that defines the relationship of thrust, fuel flow, velocity, and altitude is

$$F_n/W_f = Ae^{bV} + \beta h^2 + \delta h + z \tag{11a}$$

This being an energy balance system (*not* a force balance), the equation as it now stands is only an intermediate derivation. From Newtonian physics, it is known that integration of a force over a given displacement gives energy for a result.

Thus

$$E_T = \int F_n \mathrm{d}x = \frac{W_f A}{\bar{a}} e^{bu}\int u\mathrm{d}u$$

$$+ W_f\beta h_1^2\int \mathrm{d}x + W_f\beta 2Gh_1\int x\mathrm{d}x + W_f\beta G^2\int x^2\mathrm{d}x$$

$$+ W_f\delta G\int x\mathrm{d}x + W_f\delta h_1\int \mathrm{d}x + W_f z\int \mathrm{d}x$$

Integrating, we have

$$E_T = \frac{W_f A}{\bar{a}b}\left[V_2e^{bV} - V_1e^{bV}1\right] - \frac{W_f A}{\bar{a}b^2}\left[e^{bV}2 - e^{bV}1\right]$$

$$+ W_f\beta h_1^2 X + W_f\beta Gh_1 X^2 + W_f\beta G^2\frac{X^3}{3}$$

$$+ W_f\delta G\frac{X^2}{2} + W_f\delta h_1 X + W_f z X$$

This result, though technically correct, must be simplified by first introducing series expansions for the exponential terms. This is mandated by the fact that $\bar{a} = (V_2 - V_1)/T$. In constant velocity flight, the denominator of the exponentials would vanish, resulting in undefined terms.

Introducing the Taylor series

$$e^{bV} = \sum_{n=0}^{\infty}\frac{V^n b^n}{n!}$$

substituting and simplifying, the first term is reduced to

$$\frac{W_f AT(V_1+V_2)}{2} e^{b(V_1+V_2)/2}$$

Denoting

$$G = (h_2 - h_1)/X$$

the thrust work term reduces to

$$E_T = \int_0^X F_n\mathrm{d}x = \frac{W_f AT(V_1+V_2)}{2}e^{b(V_1+V_2)/2}$$

$$+ W_f\beta T\left[\frac{V_1+V_2}{2}\right]\left[\frac{h_1^2+h_1h_2+h_2^2}{3}\right]$$

$$+ W_f\delta T\left[\frac{V_1+V_2}{2}\right]\left[\frac{h_1+h_2}{2}\right] + W_f zT\left[\frac{V_1+V_2}{2}\right] \tag{15a}$$

Low Altitude Enhancements

The TSFC empirical relationship that was utilized in the thrust work term derivation implies a constant efficiency in the engine's capability to convert fuel energy to thrust at a constant velocity and altitude; i.e., the relationship between fuel flow and thrust is linear for a given velocity and altitude. It was recognized that the relationship did not apply at high thrust levels encountered at less than 2000 ft altitude and 250 knots airspeed, because of an increase in engine efficiency. Thus a modification was required which would provide a new thrust to fuel flow relationship above a certain thrust level (K_7). This relationship (LAM1) is defined as follows:

$$\mathrm{LAM1} = K_8 F_N + K_9$$

where F_N is derived from the original fuel burn equation, noting it is at the form

$$E_T = TV_T F_N = E_D + \Delta\mathrm{KE} + \Delta\mathrm{PE} \tag{14a}$$

Core Equation

From Eqs. (1a), (2a), (10a), (14a), and (15a) we have the equation forms to determine fuel consumption from the energy balance:

$$B = W_f T = \frac{E_D + \Delta\mathrm{KE} + \Delta\mathrm{PE}}{E_T/W_f T}$$

Thence the core equation for the fuel consumption algorithm is

$$B = T\bar{V}_T F_N/V_T P + T(\mathrm{LAM1}) \tag{16a}$$

where

$$F_N = \frac{R_1 K_1}{2} \bar{\rho} S_w \bar{V}_T^2 + \frac{2R_2 K_2 W^2}{\bar{\rho} S_w \bar{V}_T^2} + \frac{W}{gT}\left(V_{T2} - V_{T1}\right) + \frac{W}{TV_T}\left(h_2 - h_1\right)$$

$$P = K_{10} e^{K_{11}\bar{V}_T} + K_{12}\left(\frac{h_1^2 + h_1 h_2 + h_2^2}{3}\right) + K_{13}\left(\frac{h_1 + h_2}{2}\right) + K_{14}$$

$$\text{LAM1} = 0 \qquad \text{if } F_n \le K_7$$

$$= K_8 F_N + K_9 \quad \text{otherwise}$$

$$V_T = (V_{T1} + V_{T2})/2$$

In the fuel burn equation, constants K_1, K_2, and S_w define the relation between the drag and lift/weight coefficients; therefore the first two numerator terms represent the energy required to generate the change in kinetic energy (third term in the numerator containing V_{T1} and V_{T2}) and potential energy (the last term containing h_1 and h_2). The denominator terms containing K_{10}, K_{11}, K_{12}, K_{13}, and K_{14} define the relationship between fuel consumption and energy resulting from the thrust as a function of aircraft velocity and altitude. In the fuel burn equation, constants K_1, K_2, R_1, R_2, K_7, K_8, K_9, K_{10}, K_{11}, K_{12}, K_{13}, K_{14}, and S_w are aircraft specific constants and must be determined for each aircraft. The changes in aircraft drag produced by configuration changes, such as deployment of landing gear and flaps are incorporated by modifying R_1 and R_2. Based upon configuration change drag effect data, a set of empirical equations that defines R_1 and R_2 is as follows:

$$R1 = GU_1F^3 + GU_2F^2 + GU_3F + 1 \qquad \text{(gear up)}$$

$$= GD_1F^3 + GD_2F^2 + GD_3F + GD_4 \qquad \text{(gear down)}$$

$$R_2 = FDM_1F^3 + FDM_2F^2 + FDM_3F + 1$$

where F is the flap angle in degrees and GU_i, GD_i, and FDM_i are the aircraft constants.

The remaining variables such as $\bar{\rho}$ (average density over an increment calculated as a function of altitude), T (time, a path variable), V_{T1}/V_{T2} (velocity, a path variable), W (aircraft weight, a path variable), g (acceleration of gravity, a constant), and h_1/h_2 (altitude, a path variable) are generalized constants or path variables.

Additional Equations

Turbojet Fuel Flow at Idle Throttle Setting

As shown in Ref. 1, the fuel flow at idle throttle setting, minimum fuel flow, is given by the following relationship:

$$\text{Fuel flow} = \max(\text{WFMIN, WFIDLE})$$

where

$$\text{WFMIN} = L_1 e^{L_2 M^2 + L_3 M + L_4 h}$$

Minimum fuel flow produced at idle can assume a value calculated from WFMIN relationship but not less than the WFIDLE value which is constant for each engine type. This shows the effect that an aircraft minimum fuel flow cannot be reduced below a certain value due to engine operating stability considerations. This relationship defines an empirical approximation of WFMIN as a function of Mach number (M) and altitude (h).

Turbojet Fuel Flow at Maximum Thrust

As shown in Ref. 7, fuel flow at maximum thrust, maximum fuel flow, is given by

$$\text{WFLimit} = A_1 e^{A_2 M} + A_3 h^2 + A_4 h + A_5$$

This empirical relationship defines the maximum fuel flow resulting from the maximum thrust that can be generated by a turbojet engine when limited by turbine temperature maximum conditions. This limit value must be changed as a function of the three different limits that are commonly used: takeoff, climb, and cruise. This relationship represents an empirical computation of the limiting fuel flow (W_f) as a function of Mach number (M) and altitude (h). Constants A_1-A_5 are aircraft specific and are multivalued depending on whether the aircraft is operating in a limit condition of takeoff, climb, or cruise.

Time Equation

In both the turbojet and propeller forms of the fuel consumption energy balance equation, time is considered a known variable associated with the measurement of fuel consumption over an increment of a path profile. In effect, this means that an equation can be developed and utilized to generate a profile by computing the time for an aircraft to traverse a particular increment based upon certain assumed aircraft performance operating conditions. As an example, most climbs are performed at maximum climb thrust or fuel flow. Thus the WFLimit equation can be combined with the core equation (16a), which is then solved for time. In the case of a descent, the idle fuel flow relationship is used.

Next, the change in altitude (h_1 to h_2) is assumed to be a certain value, say 2000 ft; and the velocity (V_1 and V_2) can be calculated, as an example, by assuming that the aircraft is climbing at some performance condition such as at the maximum lift-to-drag ratio velocity. In any event, by this assumption, or some other, a velocity profile as a function of change in altitude can be determined. Therefore altitude, velocity, and gross weight are treated as known path profile variables with time being the unknown.

Path Profile Generation Summary

The purpose in discussing the climb and descent forms of the core equation is to demonstrate the versatility inherent in the energy balance approach to aircraft performance analysis. This, in effect, means that the equation can be manipulated to both measure the amount of fuel consumed over a defined path profile, or it can be used to generate a path profile as defined by some algorithmic policy. It is the intent that in the process of future development, an optimal policy will be developed utilizing the basic energy balance concept.

Fuel Consumption Equation Validation Process

The aircraft specific constants associated with the fuel consumption estimation core equation were derived utilizing performance manuals, noise measurement reports, and airline flight operation manuals. The data contained in these sources are intended to represent either projected aircraft performance or typical operating conditions. Also, data are not available which can be utilized to confirm the fuel estimation accuracy during all portions of climbs and descents. Therefore it is planned to utilize actual aircraft data to both calibrate and evaluate the computational accuracy of the core equation.

As part of this actual data validation effort, data have been collected onboard air carrier aircraft in the Houston area. This cockpit data consisted of 35-mm cockpit instrument pictures taken at approximately 1-min intervals during cruise, descent, landing, takeoff, and climbout to cruise. The recorded data on the pictures included fuel flow, time, gross weight, engine pressure ratio, Mach number, indicated airspeed, temperature, and altitude. Additionally, an indication of gross weight was determined by utilizing a starting gross weight in conjunction with fuel quantity gauges. Specialized operations were performed in order to evaluate the fuel consumption characteristics of aircraft being operated under these conditions. As an example, some aircraft were requested to climb and descend under conditions of maximum lift-to-drag ratios in order to evaluate the drag and thrust performance. The air traffic control enroute and terminal radar systems were also utilized to record aircraft tracking data. These data consist of aircraft ground velocity, altitude, heading, and position.

The data from these two sources will then be utilized to evaluate the accuracy of the fuel estimation. One data set will be used to perform a calibration of the aircraft specific constants for one aircraft of a particular engine/airframe type. Another data set will be used to verify the fuel estimation accuracy of the algorithm.

Of key interest in the calibration phase is the determination of the inflight aircraft drag characteristics. By rewriting the core equation, an expression can be used which contains only thrust, drag as a function of the aircraft specific constants K_1 and K_2, kinetic energy changes (velocity changes), and potential energy changes (altitude changes). The resulting equation was presented as part of the core equation (16a). By utilizing a constant engine pressure ratio during climb and looking at adjacent altitude change increments (where the altitude increments are small), a constant thrust condition exists. Since the thrust for two different increments is equal, then the two different sets of conditions, when substituted in the right side of the core equation, can be set equal. This provides a set of equations, with the only unknowns being K_1 and K_2, which yields to a simultaneous solution. In this way actual flight K_1's and K_2's can be calculated for a number of altitude and velocity conditions.

Air Traffic Control Procedural Analysis

The core equation and support equations that are described in Refs. 2, 6, and 7 have been implemented on a time sharing computer system by the FAA's Office of Environment and Energy. By means of an interactive program, the equations can be utilized by air traffic control personnel to determine estimated fuel consumption in the evaluation of the energy efficiency of air traffic control procedures.

A substantial effort has just been completed involving the Eastern Region, New England Region, air carriers, general aviation representatives, and other interested organizations aimed at identifying and determining those procedures that could be changed so as to improve fuel efficiency. As part of this study[9] 42 separate air traffic control situations were analyzed. In each situation, a new procedure was determined and the fuel savings resulting from the procedural change defined.

It is anticipated that this kind of study and resulting procedural actions could result in a savings in excess of two million gallons of fuel annually by the system users. It is also of interest to note that during the study an accuracy check was performed by region personnel in conjunction with Eastern Airlines which demonstrated model agreement with cockpit collected data by Eastern Airlines to within 3%. Even though this is not regarded as a complete validation, it is, however, indicative of the anticipated model accuracy.

Additionally, the fuel consumption model has been implemented on a microprocessor system. It has as an input device a graphics tablet which can be utilized to construct air traffic control airspace route maps and compute fuel consumption from these profiles. This system will allow field personnel to more easily evaluate the impact on fuel consumption resulting from procedural changes, in addition to presenting multiple profiles for interactive procedural analysis.

Optimization of Aircraft Path Profiles

Historically, optimal energy efficient aircraft path profiles have been generated by utilizing two basic conditions. The first condition deals with constraint of the profile in terms of certain aircraft operating parameters. As an example, during climb it would be assumed that the aircraft would climb at maximum climb thrust and that the velocity would vary in a certain prescribed or predetermined manner. This, in effect, established a set of boundary conditions or constraints on the optimal problem. The optimal path profile was then generated within these constraints. The second condition deals with separating the total path profile from departure to arrival into various segments such as climb, cruise, step climb, cruise, and descent. Each of these segments is constrained by assumptions previously mentioned. The overriding constraint is that optimal operation over each segment would produce a total optimal path profile. In other words, the sum of a group of minimum fuel burn segments would produce a total minimum fuel burn path profile. The effect of these constraints on total path profile fuel consumption is not known. However, since the fuel consumed during a total path profile is dependent on atmospheric winds and temperatures in combination with variable airframe/engine operating efficiencies, it is quite possible that a less than optimal fuel burn during a first portion of a path profile will result in additional savings later in the profile to the extent that the total fuel consumed will be reduced. This implies that improved fuel efficiency can be realized by utilizing a dynamic programming technique to generate an unconstrained total path profile that will use the minimum fuel in the presence of time or route constraints.

This approach was utilized in a FAA/NASA/MITRE project aimed at developing a general aviation flight planning aid technology for implementation on a programmable calculator or microcomputer. The first phase of this effort has been completed and is covered in Ref. 11. It remains for the technology development work to be implemented and flight tested. An additional effort is now underway to apply the concept to an air carrier environment. The concept will then be evaluated by comparison with an air carrier's existing operating methods and the potential benefits defined.

Summary

An energy balance concept has been developed and applied to aircraft energy analysis in order to define fuel conservation opportunities. This approach differs from other efforts in estimating fuel consumption. Airline calculations are intended to forecast fuel supply requirements. These are based on city pair historical use data and, therefore, lack the detail necessary for air traffic control (ATC) considerations. Air carrier flight planning is performed by utilizing an incremental table look-up routine that provides typical flight profiles. Again, the level of detail is not adequate for ATC analysis. On the other hand, simulation research uses detailed equations of motion. These contain thrust and other variables as explicit inputs. As such, they are difficult to calibrate and utilize in trajectory analysis.

The concept has been implemented as part of interactive computer based analysis aids. These aids have been used to define ATC procedural and regulatory effects on aircraft fuel consumption. Additionally, because of the simplicity of the equation form, mathematical techniques can be applied to the generation of fuel efficient total path trajectories.

Work continues in the application of the base technology in the areas of formal validation, flight planning applications and evaluations, profile generation, interface with winds and temperature data base, and definitions of fuel efficient ATC concepts.

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References

[1]Boeing Company, "Jet Transport Performance Method," Boeing Document No. D6-1420, Seattle, Wash., 1957.

[2]Collins, B.P., "Estimation of Fuel Consumption of Commercial Jet Aircraft from Path Profile Data," The MITRE Corporation, McLean, Va., MP-80W7, May 1980.

[3]Sorensen, J.A., "Concepts for Generating Optimum Verified Flight Profiles," NASA CR 159181, Sept. 1979.

[4]Erzberger, H. and Lee, H., "Characteristics of Constrained Optimum Trajectories with Specified Range," NASA TM 78519, Sept. 1978.

[5]Stengel, R.F. and Marcus, F.J., "Energy Management Techniques for Fuel Conservation in Military Transport Aircraft," AFFDL-TR-75-156, Feb. 1976.

[6]Collins, B.P., "Estimation of Turbojet Fuel Consumption from Tracking Data," The MITRE Corporation, McLean, Va., MTR-79W344, Sept. 1979.

[7]Collins, B.P., Haines, A.L., and Pool, D.A., "Derivation and Current Capabilities of the Path Profile Fuel Consumption Algorithm," The MITRE Corporation, McLean, Va., MTR-80W195, Sept. 1980.

[8]Carroll, R.L., *The Aerodynamics of Power Flight*, John Wiley and Sons, Inc., New York, 1960.

[9]"Northeast Area—Procedural Study Report," Federal Aviation Administration, Eastern Region and New England Region, Dec. 1, 1980.

[10]Winer, D.E. and Hoch, C.J., "Energy Conservation in Terminal Airspace Through Fuel Consumption Modeling," Federal Aviation Administration, Washington, D.C., paper presented at the 1980 International Air Transportation Meeting, May 1980.

[11]Collins, B.P., Haines, A.L., and Wales, C.J., "A Concept for a Fuel Efficient Flight Planning Aid for General Aviation," The MITRE Corporation, Mc Lean, Va., MTR-81W233, Jan. 1982.